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Closed-string tachyons and the Hagedorn transition in AdS space

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Abstract: We discuss some aspects of the behaviour of a string gas at the Hagedorn temperature from a euclidean point of view. Using AdS space as an infrared regulator, the Hagedorn tachyon can be effectively quasi-localized and its dynamics controled by a finite energetic balance. We propose that the off-shell RG flow matches to an euclidean AdS black hole geometry in a generalization of the string/black-hole correspondence principle. The final stage of the RG flow can be interpreted semiclassically as the growth of a cool black hole in a hotter radiation bath. The end-point of the condensation is the large euclidean AdS black hole, and the part of spacetime behind the horizon has been removed. In the flat-space limit, holography is manifest by the system creating its own transverse screen at infinity. This leads to an argument, based on the energetics of the system, explaining why the non-supersymmetric type-0A string theory decays into the supersymmetric type-IIB vacuum. We also suggest a notion of 'boundary entropy', the value of which decreases along the line of flow.

Keywords: Superstring Vacua, Black Holes in String Theory, AdS-CFT Correspondence.

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Contents

1. Introduction

The problem of the behaviour of strings at high temperature is a classic, this on account of its physical interest and its elusive aspects. The possibility that a Hagedorn spectrum [1] and its apparent limiting temperature are indications that string theory has a new phase at high temperatures has been extensively studied [2]–[7].

In the euclidean version the Hagedorn temperature can be associated with the appearence of a thermal tachyon. The world-sheet manifestation of closed-string target-space tachyons is the appearence of relevent operators which drive the system to a new infrared fixed point. In target space such a flow will have an important impact on the space. In particular it may change its dimension as the value of the Virasoro central charge decreases. The infrared fixed-point string theory should thus be described by some non-critical string background. Open-string tachyons have less of an impact; they do drive the system to a new fixed point, however in that case the infrared fixed-point system has the same value of the central charge as the ultraviolet fixed-point system. The whole system does loose degrees of freedom along the flow; these are contained in the boundary entropy function which does indeed decrease [8]–[12].

Recently an intermediate type of infrared instability was studied [13, 14, 15]. It contains a tachyon condensate somewhat localized in a non-compact space-time. The central charge does not change along the flow but the number of degrees of freedom does change and a decreasing quantity associated with the flow is identified in orbifold examples [15].

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One can be content with analyzing the end-points of the flow. One may also wish to obtain a time-dependent picture of the dynamics of the flow. In this note we set the Hagedorn transition in a time-dependent framework and offer a very qualitative picture of how the Hagedorn transition can be viewed as a flow of a 'localized' tachyon. To make this possible we will embbed the Hagedorn tachyon in an Anti de-Sitter (AdS) background which serves as a regulator. It is a non-compact space in which the temperature is redshifted as one travels along the radial direction. The redshift localizes the amount of space for which the temperatures are large enough to produce a thermal tachyon. We will describe several competing trajectories for such a flow, all of them will have a common end-point, a large stable black hole in AdS. In the process of reaching the fixed point the Hagedorn temperature region is cut out of space, i.e. the euclidean black hole is a region of space capped at its horizon. The central charge is retained in the flow as the dimension of space remains unchanged. The extent of the dimension is diminished and the amount of space gobbeled-up is related to the entropy change.

In section 2 we review and elaborate on some facts associated with the Hagedorn behaviour of strings. In particular we recall the string/black-hole correspondence [16, 17, 18], which is vindicated in this note from an euclidean point of view. In section 3 we recall mechanisms for black-hole nucleation and discuss how the AdS space can serve as regulator which will allow to expose a string/black-hole transition. In section 4 we recall the emergence and the properties of thermal tachyons. In section 5 we analyze possible trajectories along which black holes will nucleate in hot AdS and evolve to a stable black-hole configuration. In section 6 we give an argument based on energy considerations why the non-supersymmetric type-0A string background will decay into the supersymmetirc type-IIB vacuum. It will turn out that the euclidean version of this decay is nothing but the above decay of a hot AdS background into an AdS black hole. In section 7 we suggest a geometric version for the portion of the entropy that changes along the flow.

2. General aspects of string thermodynamics

The basic observation about string thermodynamics is the fact that string excitations have an asymptotic degeneracy of mass levels that increases exponentially as $\exp(\beta_s M)$ for $\beta_s M \gg 1$, where $\beta_s \sim \ell_s = m_s^{-1}$ is of the order of the string length. The Hagedorn temperature is defined as $T_s = \beta_s^{-1}$ and is naively interpreted as a limiting temperature for the string thermal ensemble.

In QCD the string spectrum mimics that of hadronic resonances, and the Hagedorn temperature T_s is associated with the deconfinement temperature. At $T > T_s$ the dynamics is better approximated by the quark-gluon constituents [19]. In the context of 't Hooft's $1/N$ expansion, one can formalize these ideas by saying that the thermodynamic entropy scales as $S = \mathcal{O}(1)$ for $T < T_s$, and $S = \mathcal{O}(N^2)$ for $T > T_s$ [20]. The possibility that a phase at $T > T_s$ may uncover fundamental degrees of freedom of critical string theory has been the main physical motivation for looking at the Hagedorn behaviour of 'fundamental' strings.

There are essentially two ways of looking at this problem. We could roughly term them as 'hamiltonian' and 'euclidean'. In the first, one works in the microcanonical ensemble, at fixed total energy, and assumes that the statistical mechanics of free strings is a good starting point, the interactions being introduced adiabatically. In the second one focuses on the formal evaluation of the canonical partition function at fixed temperature T, i.e. the euclidean path integral on $\mathcal{M} \times \mathbf{S}^1_\beta$ with a thermal spin structure and $\beta = 1/T$. Here $\mathcal{M}$ is the 'spatial' conformal field theory background, that should contain a factor of $\mathbf{R}^{d-1}$ if the strings propagate in d large spacetime dimensions (we suppose that the remaining $10-d$ dimensions are of $\mathcal{O}(\ell_s)$ length.)

In either formalism one has to face the general problem of thermal ensembles in the presence of gravity, i.e. the gravitational back-reaction of the finite energy density on the background geometry. This occurs on scales of the Jeans length. For a given total energy, the Jeans length corresponds to the radius of the equivalent Schwarzschild black hole:

$$R_J(E) = (G_d\, E)^{\frac{1}{d-3}}\,, \tag{2.1}$$

where G_d is the d-dimensional Newton constant $G_d \sim g_s^2\, \ell_s^8\, R_c^{d-10} \sim g_s^2\, \ell_s^{d-2}$. Alternatively, at fixed temperature T the g_{tt} component of the metric develops a one-loop tachyonic mass $m_J^2 \sim -G_d T^d$ (c.f. for example [21],) so that

$$R_J(T) = \frac{1}{\sqrt{G_d\, T^d}}\,. \tag{2.2}$$

In either case, we have to provide a physical 'container' with volume $V = R^{d-1} < R_J^{d-1}$ in order to maintain the framework of background flat geometry. For example, we may consider toroidal compactification of the large dimensions to a torus $\mathbf{T}^{d-1}$ of radius $R \gg \ell_s$. Beyond this point, the physics depends on the particular definition of the 'container'.

A good model for the Hagedorn spectrum is to imagine the highly excited strings as a random walk of length $\ell_s^2\, M$ [5]. One then finds that, even at the level of free strings, the gas is very sensitive to the finite-size effects introduced by the finite-volume container [6, 7]. In general there is no notion of extensivity of the multi-string thermodynamic quantities. The microcanonical density of states has the form

$$\Omega(E)_{\rm free} = \exp(\beta_s E + \cdots)\,, \tag{2.3}$$

where the dots stand for subleading contributions in the large-E limit. They can come in the form of lower powers of the energy balanced with volume factors or in terms of logarithms. Since the purely linear term in the entropy $S(E) = \beta_s E + \cdots$ gives formally infinite specific heat, the details of the thermodynamics depend on the subleading terms. In any case, within this picture the Hagedorn temperature is maximal within $\mathcal{O}(1)$ accuracy in string units. The microcanonical temperature function

$$T(E) = \left(\frac{\partial S}{\partial E}\right)^{-1} \tag{2.4}$$

grows as $E^{1/d}$ for $T < T_s$, when the gas is dominated by massless particles, and levels off at $T \approx T_s$ around $E_s = T_s(RT_s)^{d-1}$, where R is the linear extension of the finite volume of the system.

Whether the function $T(E)$ is slightly decreasing or increasing for larger energy densities depends on the subleading contributions to (2.3). These are model-dependent and can affect the physical definition of the Hagedorn temperature by numerical factors of $\mathcal{O}(1)$. On the larger picture, we will see that these details are not very important. Therefore, again within $\mathcal{O}(1)$ accuracy in string units, we can recognize the regime dominated by long strings as a plateau of the microcanonical temperature with $T \approx T_s$.

The interactions are introduced in this picture on the basis of some general physical arguments. In principle, for a fixed total volume $V = R^{d-1}$ there are two basic energy scales that depend on the string coupling g_s. First, at energy densities of order $1/g_s$,

$$E_{\rm plasma} = \frac{m_s}{g_s}\,(Rm_s)^{d-1} \tag{2.5}$$

we can nucleate D-brane anti-D-brane pairs from the thermal bath. Since D-branes are charaterized by the fundamental open strings attached to them, it is natural to assume that the *leading* linear form of the entropy $S(E) \approx \beta_s E$ is not affected. As for the subleading terms, the population of open-string modes has the general effect of tilting the curve $T(E)$ slightly upwards, i.e. open-strings work in favour of positive specific heat [7].

In principle, we cannot exclude that such relativistic plasma of D-branes might be characterized by a different density of states, perhaps dominated by the larger entropy of random p-branes with $p > 1$ (see, for example [43].) However, we consider this unlikely on the basis of the absence of a consistent quantization of relativistic p-branes with $p > 1$.

The next important scale is

$$E_g = \frac{m_s}{g_s^2}\,. \tag{2.6}$$

At this energy the entropy in the string gas is of the same order of magnitude as that of a Schwarzschild black hole:

$$S(E)_{\rm bh} = C_d\, E\,(G_d\, E)^{\frac{1}{d-3}}\,, \tag{2.7}$$

where C_d is a dimension-dependent numerical constant. Although the details of the black-hole nucleation out of the long-string gas are yet to be developed, we expect on general entropic grounds that the thermal ensemble is dominated by a single large Schwarzschild black hole for $E > E_g$. In practice, this is a generalization of the black-hole/string correspondence principle [16, 17, 18]. An idea of how this works can be obtained by a simple-minded estimate of the equilibrium between a long-string 'soup' and a black hole that nucleates out of a thermal fluctuation. The total entropy is of order

$$S(E,M) = \beta_s\,(E-M) + C_d\, M\,(G_d\, M)^{\frac{1}{d-3}}\,, \tag{2.8}$$

where E is the total energy and M is the mass in the black-hole phase. The function $S(E,M)$ decreases monotonically away from $M = 0$ and has minimum at

$$M_c \sim \frac{\ell_s^{d-3}}{G_d} \sim \frac{m_s}{g_s^2} \sim E_g\,. \tag{2.9}$$

Then, it increases monotonically for $M > M_c$. For sufficiently large total energy E, the global maximum is at $M = E$. Thus, if a black hole of mass M is nucleated by a thermal fluctuation, it will tend to evaporate for $M < M_c$. On the other hand, if $M > M_c$, it will tend to grow without bound until it engulfs the whole string gas with total available energy E. This process can be very fast, as the small black holes should nucleate everywhere and coalesce into a large black hole. Since the black holes have negative specific heat, the microcanonical temperature decreases for $E > E_g$ and we find that $T \approx T_s$ is effectively a maximal temperature of the system. Notice that this instability is physically different from the Jeans instability, since it occurs at string-scale distances.

It should be emphasized here that the status of (2.9) as a *true* minimum of the entropy functional is not solid. Since the black hole with mass M_c has a Schwarzschild radius of the order of the string scale, the entropy formula (2.7) suffers $\mathcal{O}(1)$ corrections for $M < M_c$. Thus, it is possible that there is no minimum at all and the long-string gas is unstable at even lower 'black hole' masses. On the other hand, for $M > M_c$ the formula (2.7) becomes more and more accurate and we can trust the basic picture of the growing black hole.

The growth of the black hole continues until energies of order

$$E_J = \frac{R^{d-3}}{G_d} \tag{2.10}$$

are reached, where the black hole crushes the walls of the container. This is the holographic bound for this volume, $R = R_J$, and we cannot go further in increasing the 'energy' without jeopardizing the very notion of energy by gross gravitational back reaction.

Although this picture is appealing from the point of view of the holographic principle, there is no sign of phases with temperatures substantially larger than T_s and no sign of 'dissociation' of strings into constituents (a more conventional picture arises for NCOS strings c.f. [22].) Instead, black-hole horizons shield all information about the structure of the string ensemble. In particular, the transient Hagedorn plateau at $T \sim T_s$ even dissappears for $E_s < E_g$, or

$$g_s^2 > (RT_s)^{1-d}\,. \tag{2.11}$$

Therefore, the issue of whether string constituents are uncovered is postponed until the physics of the finite-volume container is sorted out. This is precisely what the AdS/CFT correspondence gives us.

3. AdS regularization

In this section we show that a physical specification of the 'container' by means of AdS space, together with the holographic interpretation of such backgrounds, yields a global physical picture that realizes the analogy with QCD.

As a physical finite-volume regularization we take the $\mathrm{AdS}_5 \times \mathbf{S}^5$ background of radius $R = \ell_s (g_s N)^{1/4}$. Type-IIB string theory in this background with N units of RR flux through the $\mathbf{S}^5$ is conjectured to be dual to $\mathcal{N} = 4$ SYM theory on the four-dimensional

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boundary [23]. We choose a timelike killing vector so that the metric reads

$$ds^2 = -dt^2 \left(1 + \frac{r^2}{R^2}\right) + \frac{dr^2}{1 + r^2/R^2} + r^2 \, d\Omega_3^2 + R^2 \, d\Omega_5^2 \,. \tag{3.1}$$

We consider the thermal ensemble defined by $t \to -it$ and $t \equiv t + \beta$. The parameter β is the inverse temperature at $r = 0$. The local temperature $T/\sqrt{g_{tt}}$ then falls as TR/r at large $r \gg R$. On the other hand, on distance scales $\ell \ll R$ the metric is well approximated by the flat ten-dimensional metric. Therefore, hot AdS works like a 'box' or container of size R, the radius of curvature.

The flat-space limit is thus $N \to \infty$ with fixed ℓ_s and g_s. Notice that this is a difficult limit on the SYM side of the AdS/CFT duality, since it takes to infinity both N and the 't Hooft coupling $\lambda = g_s N$. The Jeans length for a string-scale temperature inside the container is

$$R_J(T = T_s) = \frac{\ell_s}{g_s} \,. \tag{3.2}$$

Therefore, we cannot take the flat space limit without bringing the Jeans instability into the interior of the 'box'. Let us suppose for the time being that $R_J > R$, so that no Jeans instability is present in the system. This forces us to tune the string coupling as $N \to \infty$ in the window:

$$\frac{1}{N} \ll g_s < \frac{1}{N^{1/5}} \,. \tag{3.3}$$

The main novelty of the infrared regulator is a change in the character of the black holes for $r_0 > R$, where r_0 is the Schwarzschild radius. In this regime, black holes engulf the 'box' and have positive specific heat, with $r_0 \sim R^2 T$, energy $E \sim N^2 \, (RT)^4/R$ and entropy $S \sim N^2 \, (RT)^3$ [24]. They mimic the high-temperature phase of a four-dimensional gauge theory. Thus, the large AdS black holes represent, via the holographic correspondence, the deconfined phase. The 'constituent' degrees of freedom being the Yang–Mills gluons of the dual theory.

For this system, the equivalent Jeans energy for the box is

$$E_J(R) = \frac{R^7}{G_{(10)}} \sim \frac{N^2}{R} \,. \tag{3.4}$$

However, larger energies are now possible, and they precisely correspond to the large AdS black holes. The curve $T(E)$ can now be continued beyond E_J, and it rises as $E^{1/4}$, exactly like a system with positive specific heat in four spacetime dimensions. Notice that a Maxwell construction suggests that a canonical ensemble analysis will jump from the massless graviton phase directly to the large AdS black hole via a first-order phase transition, skipping all the transient phases dominated by Hagedorn strings and ten-dimensional black holes [25, 26]. We have summarized the situation in figure 1.

This agrees with the canonical analysis of refs. [24, 27]. The AdS black hole dominates the thermodynamics already for $T > T_c \sim 1/R$. Since $RT_s \ll 1$ in the flat-space limit, the regime of Hagedorn temperatures cannot be studied in terms of stable canonical configurations. In [28, 29] we termed this fact as a 'Hagedorn censorship'. This means that long before the Hagedorn temperature is reached the large AdS black hole configuration

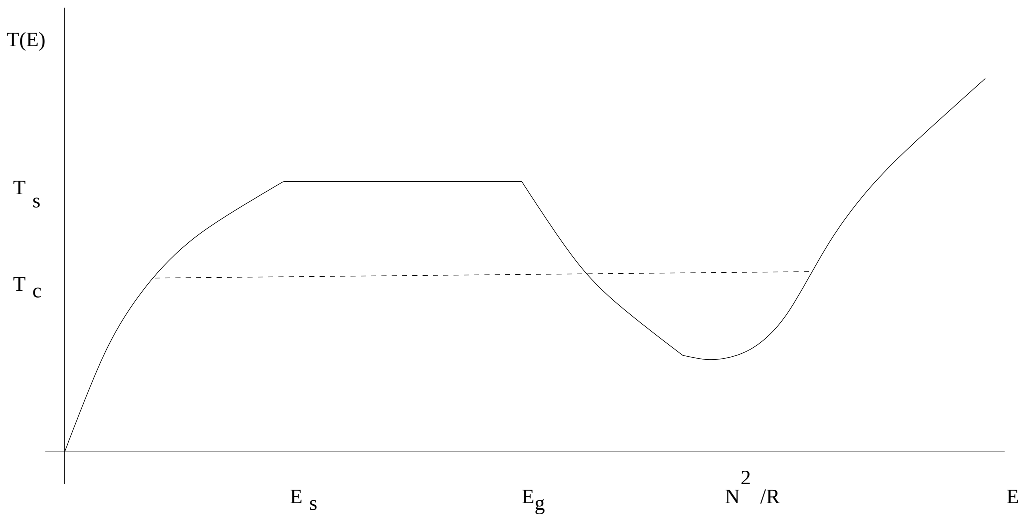

Figure 1: The microcanonical temperature as a function of energy. The Maxwell construction gives the critical temperature for the Hawking-Page phase transition at $T_c \sim 1/R$.

already dictates the asymptotic equilibrium physics. The mechanism of the transition may depend of the fate of metastable and non-stable configurations. In this note we will present a qualitative picture of this mechanism.

Even if the large AdS black hole dominates the thermodynamics above the critical temperature $T_c \sim 1/R$, it is interesting to estimate the lifetime of the AdS 'box' for $T > T_c$. For temperatures just above the critical temperature $T_c \sim 1/R \ll T_s$, the relevant physics should be understandable in low-energy terms. There are two low-energy instabilities of a massless gas in 'approximately' flat space. One is the Jeans instability and the other is the Gross-Perry-Yaffe decay by microscopic black-hole nucleation (GPY) [21]. The Jeans instability is a one-loop effect, whereas the GPY instability is a non-perturbative effect. Therefore, the Jeans instability is by far the dominant mode of decay whenever present. Let us adjust the value of the string coupling as in (3.3) so that, while having a large AdS 'box', we still have $R_J > R$ and the Jeans instability is absent. Then the dominant decay mode might be the 'sphaleron' process of GPY. These authors interpret the euclidean section of the Schwarzschild metric as a kind of sphaleron solution that gives the leading semiclassical rate for thermal nucleation of black holes. The rate is suppressed by the non-perturbative factor

$$\exp\left(-C/G_d\, T^{d-2}\right) . \tag{3.5}$$

The nucleation produces a black hole inside the box, in unstable equilibrium with the gas at temperature $T > R$. This black hole is unstable, either evaporating back into the thermal bath, or growing until it reaches the size of the large AdS black hole.

The suppression factor (3.5) may seem to be of order $\exp(-C/g_s^2)$ at temperatures of $\mathcal{O}(T_s)$. However, the calculation leading to (3.5) is not reliable at those temperatures, since the relevant saddle-point black-hole geometry has Schwarzschild radius $r_0 = \mathcal{O}(\ell_s)$. Therefore, corrections of $\mathcal{O}(1/g_s^2)$ to the exponent in (3.5) are possible by tree-level stringy effects.

As an illustration of the rapid variation of the free energy as a function of the Scharzschild radius, consider the free energy of a ten-dimensional Schwarzschild black hole of radius r_0 at temperature $T \sim T_s$. Neglecting numerical factors, it has the form:

$$\beta F = \beta M - S \sim \frac{1}{G_{(10)}}(\beta\, r_0^7 - r_0^8) \sim \frac{1}{g_s^2}\left((m_s r_0)^7 - (m_s r_0)^8\right). \tag{3.6}$$

The GPY saddle point lies at the local maximum of this function, at $r_0 = 7\ell_s/8$. However, at $r_0 = \ell_s$ we have $\beta F = 0$ and at $r_0 = 2\ell_s$ we have a large negative value $\beta F = -2^7/g_s^2$.

Because of this large sensitivity to the matching point around $r_0 \sim \ell_s$, we do not know *a priori* if the nucleation of a string-sized black hole is actually suppressed. In section 4 we shall argue that such nucleation is actually unsuppressed and appears as a *tree-level* instability of the string background.

4. Hagedorn behaviour and tachyon condensation

In the euclidean approach one works at a more formal level by studying the path integral on $X_\beta = \mathbf{S}^1_\beta \times \mathbf{T}^{10-d} \times \mathbb{R}^{d-1}$. In order to evade obvious problems with Jeans instabilities we can replace $\mathbb{R}^{d-1}$ with $\mathbf{T}^{d-1}$ of radius R, with $\ell_s \ll R \ll R_J$. The Hagedorn instability is seen here as the emergence of a tachyonic winding mode around $\mathbf{S}^1_\beta$ precisely at $\beta < \beta_s$ [3, 4]. The singularities of the thermodynamic functions are dominated by the infrared singularities induced by the tachyon in string diagrams.

In this picture, there is no formal objection to taking $T > T_s$, except for the existence of the tachyonic mode. Therefore, if one could solve for the dynamics of the tachyon condensation, one would have a picture of the $T > T_s$ phase. In a mean-field Landau–Ginzburg approach one would seek a static effective action for the tachyon field σ that gives the following contribution to the euclidean action or free energy:

$$I_\sigma = \frac{\beta_s}{2G_d}\int d^{d-1}x\ \left[f(|\sigma|)\,|\partial\,\sigma|^2 + m_s^3\,(\beta-\beta_s)\,|\sigma|^2 + m_s^2\,V(|\sigma|) + \cdots\right]. \tag{4.1}$$

Notice that both positive and negative winding modes contribute, making the Hagedorn tachyon into a complex scalar field. The dots stand for higher derivative terms generated at tree level or other contributions generated at higher loops or even non-perturbatively (c.f. for example [30].)

It was argued in [4] that the interaction with the dilaton affects $V(|\sigma|)$ so that one has actually a first-order phase transition at a slightly smaller temperature than T_s. In any case, since $V(|\sigma|)$ is already non-trivial at the tree level, the dynamics of the tachyon destabilizes the system very fast, and the quasi-static approximation is not appropriate. In particular, free energies of $\mathcal{O}(1/g_s^2)$ are released in the process of condensation.

A standard technical assumption is that the real-time dynamics of the tachyon condensation process might be mimicked by some renormalization-group (RG) flow on the world-sheet. Thus, we speak of RG flow metaphorically, as if it was real time evolution, but we just have a collection of off-shell string backgrounds. For example, the failure to satisfy the tachyon equation of motion is just the beta function for the corresponding world-sheet coupling:

$$\beta_\sigma \propto \frac{\delta I}{\delta \sigma}\,. \tag{4.2}$$

One picture for the required RG flow was developed in the early days in terms a Kosterlitz-Thouless transition on the world-sheet [3]. In this picture the tachyon vertex operator is a world-sheet vortex and its condensation renders the euclidean-time world-sheet field massive. Since one dimension 'dissapears' it appears that the end-point of condensation must be a non-critical string with a Liouville field taking the role of 'time', as in [39]. Although this seems like the natural set-up for the real-time description of the condensation process, the lack of a clear thermodynamical interpretation of the end-point is rather puzzling.

At any rate, the end-point of the condensation of the Hagedorn tachyon is guaranteed to have a non-trivial physical interpretation, if only because the tachyon itself is not a physical state of the string theory at zero temperature. Thus, it is not clear even in principle what the expectation value of the σ-field means. This is somewhat reminiscent of similar situations in the study of the decay of non-BPS branes to a pure closed-string vacuum [32]: one needs some dynamical mechanism, presumably operating at the tree level, to suppress unwanted excitations at the condensed vacuum. In the case of the Hagedorn tachyon, such a requirement could be incorporated by proving that the kinetic function $f(|\sigma|)$ in (4.1) has a zero at the end-point of tachyon condensation.

One striking result is that of refs. [41]. In these works the tachyon potential, together with other moduli, is argued to be calculable on the basis of supersymmetry constraints. Since one finds (euclidean) vacua with restored supersymmetry, it is natural to view them as end-points of tachyon condensation in close analogy with the behaviour of localized tachyons on D-branes or orbifolds [32, 13]. This view is also vindicated by recent conjectures on the fate of the closely related type-0 string backgrounds [33]. These would decay into supersymmetric type-II string backgrounds [36, 40, 31].

In any case, a decay of the canonical ensemble geometry into a background with supersymmetry (typically flat space or some domain wall solutions) is also difficult to interpret in terms of the original statistical mechanical problem, especially in view of the picture outlined before in the microcanonical ensemble.

In view of the previous remarks, perhaps the most striking fact about the canonical approach is the lack of a clear physical picture. In fact, it offers no obvious candidate for realizing the 'QCD scenario', i.e. the liberation of 'string constituents'. In this paper we make an attempt at providing such a physical picture.

5. Euclidean black holes and the matching hypothesis

The idea that closed-string tachyons destabilize the vacuum and drive it into a new vacuum has appeared in the past in various settings. In the simplest form a spacetime coordinate becomes massive on the world-sheet and disappears in the deep infrared leading to an effective reduction of the dimension of spacetime [3], or crystalization and formation of

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asymptotic curvature [38]. While this flow has a certain appeal its main drawback is that it forces us into the setting of a non-critical string theory in which little progress has been made so far. However one may consider a situation where the effect on space of the flow is important but less drastic. The dimensionality of space does not change in the infrared but nevertheless the extent of this dimension is reduced. This can happen in a setup of localized tachyons whose flow does not change the bulk central charge but does does change some parameter counting a lower dimensional change in the degrees of freedom [13, 15].

In this section we argue that such ideas find a natural implementation in the AdS regularization of the Hagedorn behaviour. Let us consider the $\mathrm{AdS}_5 \times \mathbf{S}^5$ thermal background at temperature slightly above the Hagedorn limit $\beta < \beta_s$. For $r < R$ we can consider the thermal manifold as well approximated by $\mathbf{S}^1_\beta \times \mathbb{R}^9$. On the other hand, for $r > R$ the temperature drops below the string scale, so that thermal winding modes are never tachyonic in the asymptotic region. Therefore, we can use the AdS/CFT background as an infrared regulator that regards the Hagedorn tachyon as a 'quasilocalized' tachyon.

For this quasilocalized tachyon there is an obvious generalization of the picture developed in [13]. Since spacetime for $r \ll R$ is an approximate cylinder with a slightly r-dependent opening angle, it is natural to suppose that it behaves similarly to a narrow cone such as those obtained from a $\mathbb{C}/\mathbb{Z}_n$ orbifold for large n. In that case, there would be an effective capping of the cone propagating towards large r. In our case we are actually looking for static euclidean interpolating backgrounds that decrease the free energy. Hence, it is natural to assume that the appropriate configurations are a set of backgrounds labelled by $r_c > 0$, with an effective string-scale capping around radial coordinate $r = r_c$. In principle, the free energy of such backgrounds should be defined from the world-sheet CFT description, or perhaps in the holographic dual SYM theory.

A more formal statement would be that there is a field redefinition with the property that the tachyon expectation value is nothing but the value of the radial variable r_c at which the geometry is capped:

$$|\sigma| \sim m_s \, r_c \,. \tag{5.1}$$

The decay of the tachyonic part of the manifold proceeds primarily via a tree-level RG flow on the world-sheet. For example, one interesting concrete mechanism is some analogue of the Kosterlitz-Thouless flow of refs. [3]. For inverse temperature $\beta < \beta_s$, the r-dependent size of the thermal circle $\mathbf{S}^1$ is

$$\beta(r) = \beta \sqrt{1 + r^2/R^2} \,. \tag{5.2}$$

As a rough estimate of the capping coordinate, we may define it by $\beta(r_c) = \beta_s$. In the flat-space limit $R \gg \beta_s$ this yields

$$r_c(\beta) = R \sqrt{\left(\frac{\beta_s}{\beta}\right)^2 - 1} \,. \tag{5.3}$$

This formula shows that the capping distance can be kept smaller than $\mathcal{O}(R)$ only in a scaling limit. Namely, unless $\beta_s - \beta$ is fine-tuned to be very small, the stringy capping occurs around $r_c = \mathcal{O}(R)$, i.e. the radius of the AdS/CFT 'box'. In order to estimate the free energy released in this process we need a concrete geometrical model for the spacetime decay.

The euclidean section of a black-hole spacetime has an effective capping of the geometry at the radial coordinate of the horizon. From the discussion in section 2, we know that the process of black-hole nucleation and growth is expected to play an important role in the exit from the Hagedorn regime. Given these two facts, it seems very natural to regard the euclidean black-hole geometry as an effective geometrical model for the capping of the thermal cylinder.

Therefore, our main hypothesis is simply that the stringy geometry resulting from the off-shell tachyon dynamics with $\sigma \sim m_s r_c$ matches onto an euclidean AdS black-hole geometry with Schwarzschild radius

$$r_0 \sim r_c(\beta)\,. \tag{5.4}$$

The black-hole geometry with horizon at $r = r_0$ is:

$$ds^2 = \mu(r_0, r)\, dt^2 + \frac{dr^2}{\mu(r_0, r)} + r^2\, d\Omega_3^2 + R^2\, d\Omega_5^2\,, \tag{5.5}$$

where

$$\mu(r_0, r) = 1 + \frac{r^2}{R^2} - \left(1 + \frac{r_0^2}{R^2}\right) \frac{r_0^2}{r^2}\,. \tag{5.6}$$

Our choice of an AdS black hole rather than a ten-dimensional Schwarzschild black hole is motivated by our desire of maintaining the SO(6) symmetry that is inherited from the $\mathbf{S}^5$ factor. This is a symmetry of the initial conditions, since we were assuming uniform thermal ensemble on $\mathrm{AdS}_5 \times \mathbf{S}^5$. The dynamical capping of the geometry may proceed by initial nucleation ten-dimensional black holes that 'coalesce' into a uniform AdS black hole with SO(6)-symmetric horizon. As a technical simplification, we consider only the SO(6)-symmetric manifolds (5.5) for which we have an explicit analytic form.

Notice that, according to the remarks in section 3, the GPY 'sphaleron barrier' (3.5) suffers from an $\mathcal{O}(1)$ ambiguity at string-scale temperatures. In fact, our mechanism suggests that at $T \geq T_s$ the barrier is completely washed out by the tree-level dynamics of the Hagedorn tachyon (4.1). On the other hand, at temperatures not too far *below* T_s, it is tempting to say that the Atick–Witten first-order phase transition described in [4] and the GPY nucleation are one and the same physical process.

We now take (5.5) as a model for the capped geometry and investigate the euclidean free energy of these manifolds.

5.1 Off-shell black holes

The free energy of the AdS black hole reads (in the following we rely heavily on formulas from [27]):

$$\beta\, F = \beta\, M - S\,, \tag{5.7}$$

where M is the mass of the black hole:

$$M = \frac{3\mathrm{Vol}(\mathbf{S}^3)}{16\pi G} \left(\frac{r_0^4}{R^2} + r_0^2\right), \tag{5.8}$$

normalized to the AdS vacuum manifold, and S is the Bekenstein–Hawking entropy:

$$S = \frac{\mathrm{Vol(Horizon)}}{4G} = \frac{\mathrm{Vol}(\mathbf{S}^3)}{4G}\, r_0^3\,. \tag{5.9}$$

The free energy can be computed as the difference of euclidean actions between the black hole manifold X_{r_0} and the thermal AdS manifold X_0:

$$\beta F = \Delta I(r_0) = I\left[X_{r_0}\right] - I\left[X_0\right]. \tag{5.10}$$

The euclidean action has the form (c.f. [42]):

$$I(X) = -\frac{1}{16\pi G}\int \left(\mathcal{R} + \frac{6}{R^2}\right) - \frac{1}{8\pi G}\int_{\partial X} \mathcal{K}\,, \tag{5.11}$$

where we use the effective five-dimensional gravity action, with $G \sim g_s^2 \ell_s^8 / R^5$. These formulas should provide a good quantitative estimate of the free energy for black holes that are sufficiently large in string units: $r_0 \gg \ell_s$.

Notice that the black-hole capping changes the topology of the space-time manifold. While X_0 is homeomorphic to $\mathbf{S}^1 \times \mathbb{R}^4 \times \mathbf{S}^5$, the topology of X_{r_0} is $\mathbb{R}^2 \times \mathbf{S}^3 \times \mathbf{S}^5$. In fact, X_{r_0} with metric (5.5) is not a solution of the equations of motion in general, because it has a conical singularity at $r = r_0$ with deficit angle:

$$\delta = 2\pi \left(1 - \frac{\beta}{\beta(r_0)}\right), \tag{5.12}$$

where $\beta(r_0)$ is the inverse Hawking temperature of a black hole with radius r_0:

$$\beta(r_0) = \frac{2\pi R^2 r_0}{2r_0^2 + R^2}\,. \tag{5.13}$$

The conical singularity contributes a finite amount to the euclidean action of the black hole:

$$\Delta I = \Delta I_{\mathrm{bulk}} + I_\delta\,. \tag{5.14}$$

The contribution proportional to the deficit angle comes from a delta-function singularity in the curvature scalar and is proportional to the themodynamic entropy:

$$I_\delta = -\delta\, \frac{S}{2\pi}\,. \tag{5.15}$$

The bulk contribution is proportional to the volume difference between the two manifolds

$$\Delta I_{\mathrm{bulk}} = \frac{1}{2\pi G}\, \left[\mathrm{Vol}\left(X_{r_0}\right) - \mathrm{Vol}\left(X_0\right)\right]. \tag{5.16}$$

In evaluating (5.16), there is a subtlety in that the volume of the black-hole manifold X_{r_0} must be computed with a rescaled metric that takes into account the slight difference in the euclidean-time periods, when compared at large cutoff radial scale L. Thus, the volume of the black-hole manifold comes with an extra factor of

$$\frac{\beta'}{\beta} = \sqrt{\frac{\mu(0,L)}{\mu(r_0,L)}} = 1 + \frac{R^2}{2}\left(1 + \frac{r_0^2}{R^2}\right)\frac{r_0^2}{L^4} + \cdots\,. \tag{5.17}$$

Although this is a negligible contribution to the action *density* in the limit where we remove the cutoff $L \to \infty$, it does contribute to the volume integral a finite term. This finite term is not very important for $r_0 \gg R$, but it is of relative $\mathcal{O}(1)$ for $r_0 < \mathcal{O}(R)$.

For a given temperature $T = 1/\beta$, larger than the critical temperature T_c, there are two stationary points at which $\delta = 0$. They correspond to the two roots of the equation

$$\beta = \frac{2\pi R^2 r_0}{2r_0^2 + R^2}\,. \tag{5.18}$$

For $\beta \ll R$, we have a small AdS black hole with $r_0 = \mathcal{O}(\beta)$ and a large AdS black hole with $r_0 = \mathcal{O}(R^2 T)$. The small black hole corresponds to a local maximum of ΔI and it is unstable, with negative specific heat. The large AdS black hole corresponds to the global minimum of ΔI and it is stable, with positive specific heat.

All off-shell black holes with $r_0 < \mathcal{O}(\beta)$ and $r_0 > \mathcal{O}(R^2 T)$ are hotter than the thermal radiation and thus evaporate back to lower radii. On the other hand, all off-shell black holes with $\mathcal{O}(\beta) < r_0 < \mathcal{O}(R^2 T)$ are cooler than the radiation and tend to grow. The coolest black hole has $r_0 = R/\sqrt{2}$ and deficit angle

$$(\delta)^{\max} = 2\pi - \frac{\beta}{R}\left(\sqrt{2} + \frac{2}{\sqrt{2}}\right). \tag{5.19}$$

For large deficit angles the geometry is that of a very narrow cone that starts at $r = r_0$ and has length of $\mathcal{O}(r_0)$. At this point it merges into a smooth geometry with curvature of $\mathcal{O}(1/R^2)$. Note that the part of the free-energy release that comes from the progressive 'opening' of the cone can be literally described in terms of the mechanism in [13]. The evidence from [37] is that such stringy decay of the cone respects the semiclassical form of the thermodynamic functions, provided $r_0 \gg \ell_s$.

We notice however that in this problem one cannot simply model the black-hole manifold as a flat cone. In fact, the balance between the conical (5.15) and bulk (5.16) contributions to the free energy is such that for small off-shell black holes $r_0 < R$, the complete free energy decreases while the deficit angle actually *increases*. It is only for $r_0 > \mathcal{O}(R)$ that both contributions to the free energy decrease in the process of black-hole growth.

5.2 The semiclassical regime of the tachyon roll

In order to trust the low-energy formulas for the free energy we must have a matching point $r_0 \sim r_c(\beta) \gg \ell_s$. Since $\beta \sim \ell_s$ and $\ell_s \ll r_c(\beta) < \mathcal{O}(R)$, we find that the deficit angle at the matching is approximately maximal

$$\delta_c \approx 2\pi\,. \tag{5.20}$$

This means that the stringy process of decay matches at $r_c(\beta)$ to a very off-shell black hole that is very unstable towards growth, so that the RG flow continues semiclassically. The amount of free energy released at the moment of the matching is of order:

$$\Delta I_{\text{matching}} \sim -\frac{r_c^3}{G}\,. \tag{5.21}$$

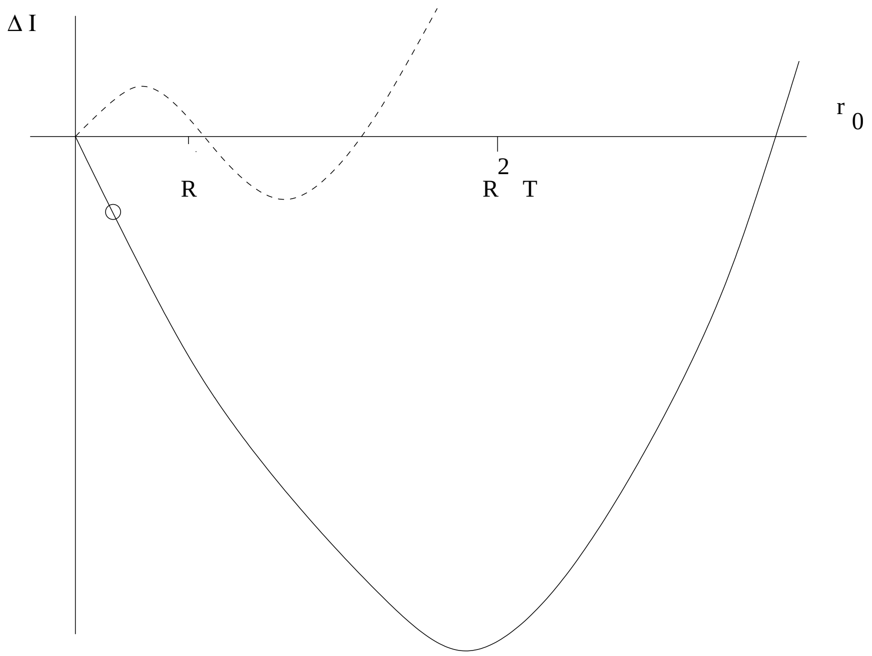

Figure 2: Schematic plot of the free energy of AdS black holes. The dashed line is the free energy at temperatures slightly above the critical temperature $T_c = \mathcal{O}(1/R)$. The local maximum is the small AdS black hole, which is the GPY saddle point for this space, and the global minimum is the large AdS black hole. The full line gives the free energy at $T > T_s \gg 1/R$. The circle indicates the matching point where string effects become dominant. It is assumed that these stringy effects wash out the GPY saddle point and at $r_0 \sim 0$ one sees the tree-level instability of the Hagedorn tachyon.

For $r_c = \mathcal{O}(R)$, this yields

$$\Delta I_{\text{matching}} \sim -\frac{R^8}{g_s^2 \ell_s^8} \sim N^2 \,. \tag{5.22}$$

In the semiclassical stage of the flow, for $r_0 > r_c(\beta)$, the beta function of the tachyon is proportional to the deficit angle:

$$\beta_\sigma \propto \frac{dI}{dr_0} = -\frac{3\mathrm{Vol}(\mathbf{S}^3) r_0^2}{8\pi G}\, \delta \,. \tag{5.23}$$

Therefore, the RG flow continues until the black hole grows into a large, stable AdS black hole at at $r_0 \sim R^2 T \sim \ell_s \sqrt{g_s N}$. At this point the deficit angle vanishes and so does the beta function.

Thus, our physical picture is the following. For $1 \ll TR \ll T_s R$ the only channel for decay into the large AdS black hole is the GPY nucleation, a process that is exponentially suppressed (we are assuming that the scales were chosen so that there is no Jeans instability.) Hence, even if the large AdS black hole is the dominant thermodynamical configuration, the vacuum AdS manifold is still long-lived for small string coupling.

As the Hagedorn temperature is approached, the GPY barrier starts to feel the string α' corrections. We conjecture that the free energy as a function of the black-hole radius matches to the tachyon potential, so that the first-order phase transition described in [4] would be simply the GPY nucleation when continued into the stringy domain. Finally, for

$T \geq T_s$, the GPY barrier dissappears and the Hagedorn tachyon condenses via a tree-level (and therefore fast) process. The decay can be followed semiclassically for $r_0 > r_c(\beta)$ as the growth of a cool black hole in a hotter medium. Thus, at the Hagedorn temperature, the formation of the large AdS black hole becomes a classical process and the thermal AdS manifold is short-lived. We summarize the situation in figure 2.

5.3 The end-point of the Tachyon condensation

The main prediction of this scenario is that the expectation value of the tachyon at the true minimum is

$$\langle |\sigma| \rangle \sim m_s \, r_0 \sim \sqrt{g_s N} \,. \tag{5.24}$$

This is hierarchically larger than the value of the tachyon at the end of the 'stringy' condensation process, which was $\mathcal{O}(m_s r_c) = \mathcal{O}((g_s N)^{1/4})$. Also, the total free energy released in the complete process is of order

$$\Delta I_{\rm total} \sim -\frac{R^6 T^3}{G} \sim -N^2 \, (RT)^3 \,. \tag{5.25}$$

The possibility of having *bona fide* σ-field fluctuations around the true vacuum can also be addressed here. Since the supergravity interpretation of the σ zero mode is the horizon location, the frequency of oscillations of the stable end-point is proportional to $d^2 I/dr_0^2 \sim R^2/G$, when calculated around the stable minimum. This diverges in the flat space limit $N \to \infty$ at fixed ℓ_s and g_s. Thus, in the flat-space limit the σ fluctuations are frozen in the stable vacuum.

Finally, the apparent emergence of supersymmetric vacua as candidate end-points of tachyon condensation can be given a simple thermodynamic interpretation in our model. The basic observation is that the euclidean manifold corresponding to the final AdS black hole is simply connected. Therefore, the spin structure is unique and at the horizon fermions are locally smooth and in particular periodic in the natural polar coordinate centered at $r = r_0$. This is a general feature of any euclidean black-hole metric in a supersymmetric theory, i.e. black holes are always 'supersymmetric' in the vicinity of the horizon capping.

Now, in the flat space limit $N \to \infty$ the curvature at the horizon of the large AdS black hole goes to zero and we are left with a better and better approximation of $\mathbb{R}^{10}$ with the standard spin structure. Thus, in this limit we would interpret the end-point of the decay process as supersymmetric (euclidean) type-IIB spacetime, but sitting at the transverse boundary of the original spacetime.

This is a remarkable manifestation of holography which could have important applications to other patterns of decay of non-supersymmetric backgrounds. In the next section 7 we outline the first such application.

6. The decay of 0A into IIB

The previous argument about the local supersymmetry of the large AdS black hole around the horizon can be used to give an elementary proof of the decay of a type-0A string theory into a supersymmetric type-IIB model. More exactly, we establish the decay of the corresponding Wick-rotated theories in the sense that we have discussed in the paper.

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The basic point is that the type-0A theory is formally defined as the high-temperature limit of IIB theory, via standard T-duality [35, 4, 34]:

$$\mathbf{0A} = \lim_{\beta \to 0} \mathbf{IIB}_\beta \,, \tag{6.1}$$

where $\mathbf{IIB}_\beta$ stands for the type-IIB theory on $\mathbf{S}^1_\beta \times \mathbb{R}^9$ with thermal boundary conditions. But we just argued that the decay of $\mathbf{IIB}_\beta$ can be regularized as the decay of $\mathrm{AdS}_5 \times \mathbf{S}^5$ at high temperature. In fact, even if our picture of the tree-level flow is not right, there is little doubt that this background decays into the large AdS black hole, a consequence of holography. But then, either in the flat-space limit of AdS or in the large-temperature limit, the euclidean black-hole manifold cannot be locally distinguished from flat $\mathbf{R}^{10}$. The RR field-strength through $\mathbf{S}^5$ scales as $F_{\mathrm{RR}} \sim N/R^5$. Thus, it vanishes locally as $N^{-1/4}$ in the flat-space limit.

Therefore, we have shown that indeed the Wick-rotated type-0A background does decay into the type-IIB supersymmetric background. If we now perform a Wick rotation back to lorentzian signature in some spectator dimension, (different from the compact supersymmetry-breaking dimension of the type 0A) we have a similar statement about the real-time theories.

Other arguments in favour of this decay have appeared in the literature, [40, 31]. We believe that the main advantage of our argument is that it provides a clear regularization that also gives an interpretation in terms of energy balance. In some sense, such regularization is necessary in order to be able to pose the question in precise terms.

7. Boundary central charge and local central charge

In [15] a notion of 'boundary central charge' was introduced in the study of orbifold tachyon condensation. It is supposed to play a role similar to that of the boundary entropy [8, 9, 10, 11, 12] in open-string boundary RG flows. Namely, it should measure the decrease in the effective number of degrees of freedom along the RG flow, in those cases where the central charge does not change. The formal definition of the boundary central charge g_{cl} is in terms of the twisted torus partition function on the orbifold:

$$\lim_{\tau_2 \to 0} \; \mathcal{Z}(\tau, \bar{\tau})_{\mathrm{twisted}} \propto g_{cl} \, \exp(\pi c / 6\tau_2) \,. \tag{7.1}$$

After factoring out by the geometric volume of the orbifold fixed point, the quantity g_{cl} is extracted:

$$g_{cl} = \frac{1}{|\Gamma|} \sum_{g \neq 1} \frac{1}{N_g} \,, \tag{7.2}$$

where N_g is the number of fixed points in the sector twisted by the group element g and Γ is the orbifold discrete group. The authors of [15] have computed this quantity explicitly for various orbifold CFTs and find agreement with the conjecture that g_{cl} decreases along RG flows. In particular, for $\mathbb{C}/\mathbb{Z}_n$ one gets

$$g_{cl}(n) = \frac{1}{12} \left(n - \frac{1}{n} \right) \,, \tag{7.3}$$

which indeed increases with n. Thus, such g_{cl} would decrease for the flows in [13] that decrease n.

We can give a physical interpretation of (7.3) as follows. From the original definition (7.1) we see that g_{cl} has a factor of the available volume for propagation of twisted strings in target space. Since we are factoring out the trivial geometrical volume of the fixed-point submanifold, there remains the effective volume given by the spread of the wave function of twisted states in the directions transverse to the fixed-point submanifold.

For the case of a cone, we can estimate the wave-function support of the lowest-lying tachyonic state by noticing that a twisted string is a winding mode around the tip of the cone. The ground-state string (non-oscillating) is forced to be localized near the tip because dragging it out costs at least the static energy of the wrapping around the base of the cone. If we consider a cone of deficit angle $2\pi(1-1/n)$ with base-length of $\mathcal{O}(\ell_s)$, the volume is $\pi\ell_s^2\, n$. Thus, this argument captures the leading large-n asymptotics of (7.3).

Therefore, we propose a physical interpretation of g_{cl} as a measure of 'depleted volume' by the condensation process or, in other words, a measure of the extension of the capping.

For the case of the thermal manifold $\mathbf{S}^1 \times \mathbb{R}^9$, the analogue of the twisted partition function (7.1) is the integrand of the one-loop free energy in the sectors with non-trivial thermal winding. Therefore, the analogue of g_{cl} is given by effective volume available for winding modes, in this case all $\mathbf{R}^9$. However, for the case of thermal $\mathrm{AdS}_5 \times \mathbf{S}^5$, thermal winding modes are effectively confined to the 'box' of radial extent $r < \mathcal{O}(R)$. For a capped manifold at $r = r_c$ we have, within $\mathcal{O}(1)$ accuracy in string units:

$$\mathrm{Vol}(X)^R_{r_c} = \frac{1}{4}\,\beta\,\mathrm{Vol}(\mathbf{S}^3)\,\mathrm{Vol}(\mathbf{S}^5)\,R^5\,(R^4 - r_c^4)\,. \tag{7.4}$$

By analogy with the case of the cone, we may divide by the volume of the 'box' orthogonal to the (t,r) space, which is of $\mathcal{O}(R^8)$. Thus, we obtain the following quantity in string units:

$$g_{cl} \sim (g_s N)^{1/4}\left(1 - \frac{r_c^4}{R^4}\right). \tag{7.5}$$

This expression is valid for $\ell_s \ll r_c < \mathcal{O}(R)$. The overall scale for such boundary central charge is set by the radius of curvature of the AdS space in string units. It is clearly decreasing with the RG flow if this is interpreted as the extension of the capping to larger radii. For $r_c > R$ the value of g_{cl} should decay exponentially, since the thermal winding modes become very massive in this region.

The information about the RG flow contained in g_{cl} comes from a one-loop diagram in closed-string theory. In fact, it is related to the one-loop correction to the free energy. The analogous quantity at tree level is thus the classical free energy that we have studied in the previous section 7. In this sense, it is very interesting that at least the bulk contribution to ΔI is essentially given by a measure of the 'depleted volume':

$$\Delta I = I_\delta + \frac{1}{2\pi G R^2}\left[\mathrm{Vol}(X_{r_0}) - \mathrm{Vol}(X_0)\right], \tag{7.6}$$

Although ΔI is monotonically decreasing between the small and large stationary black holes, for $\beta \ll r_0 \ll R^2 T$, neither the bulk piece nor the conical singularity contribution are separately monotonic in this range.

The bulk contribution is actually monotonically *increasing* for $\mathcal{O}(\beta) < r_0 < \mathcal{O}(R)$, because of the offset volume effect at large radius of eq. (5.17). On the other hand, I_δ reaches its minimum at $r_0 = \mathcal{O}(R^2 T)$ but at a value strictly smaller than the Schwarzschild radius of the large AdS black hole. This is easy to understand since $I_\delta = 0$ at the large stable AdS black hole.

It might be interesting to consider alternative splittings of the euclidean action. For example, instead of distinguishing between the volume contribution and the conical singularity, we can split the black-hole manifold in two pieces by cutting out a small cone D_ϵ of radius ϵ around the horizon in (r,t) space. Thus, $X = X_\epsilon + (X - X_\epsilon)$ with $X_\epsilon = D_\epsilon \times \mathbf{S}^3 \times \mathbf{S}^5$. Then, the contribution to ΔI from X_ϵ (with due attention to the contribution of the extrinsic curvature term to the action,) gives minus the thermodynamic entropy:

$$- S = \lim_{\epsilon \to 0} \left[-\frac{1}{16\pi G} \int_{X_\epsilon} \left(\mathcal{R} + \frac{6}{R^2} \right) - \frac{1}{8\pi G} \int_{\partial X_\epsilon} \mathcal{K} \right] . \tag{7.7}$$

The remaining piece of the action computes the AdS analogue of the ADM mass:

$$\beta\, M = \lim_{\epsilon \to 0} \left[-\frac{1}{16\pi G} \int_{X - X_\epsilon} \left(\mathcal{R} + \frac{6}{R^2} \right) - \frac{1}{8\pi G} \int_{\partial(X - X_\epsilon)} \mathcal{K} \right] . \tag{7.8}$$

Of course, the expression for the mass must be normalized by that of the AdS vacuum. In the process one learns that the boundary term at infinity gives no contribution. However, the boundary term at the horizon does give an important positive contribution which makes M grow with r_0, despite the fact that the bulk action is just measuring the available volume and this is shrinking.

Then we can normalize the free energy by the volume of the space orthogonal to the (r,t) plane and define the classical contribution to the boundary central charge as:

$$\mathcal{G}_{cl} = \frac{\Delta I}{R^5\, \mathrm{Vol}(\mathbf{S}^5)\, \mathrm{Vol}(\mathrm{Horizon})} . \tag{7.9}$$

The previous splitting induces two contributions to the central charge:

$$\mathcal{G}_{cl} = \mathcal{G}_{cl}^{\mathrm{cone}} + \mathcal{G}_{cl}^{\mathrm{ADM}} , \tag{7.10}$$

where the cone contribution is constant,

$$\mathcal{G}_{cl}^{\mathrm{cone}} = \frac{1}{8\pi G} , \tag{7.11}$$

and the 'mass' contribution is monotonically decreasing for $r_0 < R$:

$$\mathcal{G}_{cl}^{\mathrm{ADM}} = \frac{3\beta}{16\pi G} \left(\frac{r_0}{R^2} + \frac{1}{r_0} \right) . \tag{7.12}$$

This is analogous to the boundary central charge in open-string theory (which also appears as a classical contribution to the effective action of open strings.) The 'tension-like' contribution to the boundary central charge decreases as long as the space-time defect is governed by approximately flat-space energetics, i.e. for $r_0 < R$.

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To conclude, we notice that one can define a notion of 'local central charge' as the integrand of the euclidean action. This is based on the fact that, expanding around flat space, a central-charge deficit is seen in the low-energy effective action as a tree-level cosmological constant. Thus, if we write

$$\Delta I = \frac{1}{20\pi G R^2} \int d\mathrm{Vol}\ \left[C(r) - 10\right], \tag{7.13}$$

we can regard $C(r)$ as a local measure of the number of bosonic dimensions. In particular $C(r) \to 10$ as $r \to \infty$. We find,

$$C(r) = C_\delta\ \delta(r - r_0) + 10\ \Theta(r - r_0)\,, \tag{7.14}$$

where Θ is the step function and we neglect the cutoff effect (5.17) that would sligtly increase the asymptotic dimensionality of space. Thus, we see that the smooth part has $C(r) = 0$ for $r < r_0$, which corresponds to the fact that this part of space-time has dissappeared behind the horizon of the black hole. The delta-function contribution is proportional to I_δ and negative:

$$C_\delta = -10\, G\, R^2\, S\ \delta\,. \tag{7.15}$$

In our previous interpretation of I_δ we should think of this term as being smoothed out by short-distance effects in such a way that the total action is still well approximated by the integral (7.13). Thus, we should have a smooth transition around $r \sim r_0$ between $C = 0$ and $C = 10$ spacetimes.

8. Conclusions

In this note we have developed a physical picture for the decay of hot type-IIB strings in approximately flat ten-dimensional spacetime. In particular, we have investigated the geometrical interpretation of the condensation of the Hagedorn tachyon.

We make use of the AdS/CFT regularization in order to control the infrared problems posed by a tachyonic vacuum. In this fashion we make the Hagedorn tachyon into a 'quasilocalized' tachyon, similar to those arising at fixed points of nonsupersymmetric orbifolds.

We propose that the dynamics of the Hagedorn tachyon induces a progressive capping of the thermal manifold in such a way that the part of space with local temperature above T_s dissappears. This picture is natural from various points of view: the geometrical picture is directly suggested by the euclidean continuation of the process of black-hole growth that is expected on the grounds of the string/black-hole correspondence; it is also suggested by similar situations in the decay of nonsupersymmetric orbifolds [13].

The mechanism for the capping could be the some version of the Kosterlitz-Thouless RG flow. We notice that, as the effects of the infrared regulator are felt, i.e. when $r_0 \sim R$, the decay can be described semiclassically as the growth of a large AdS black hole that is cooler than the thermal radiation outside it. This continues until the stable AdS black hole is reached.

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If the regulator is removed, the stable euclidean manifold of the AdS black hole approaches $\mathbb{R}^{10}$ with the standard spin structure. Therefore, there is a sense in which the string ensemble decays into a supersymmetric vacuum. It is just the euclidean geometry of Rindler space! An immediate consequence of this phenomenon is the conjectured decay of type 0A unstable vacuum into the type-IIB vacuum. Here the result has been established for the Wick-rotated theories.

We have also made a preliminary attemp at defining a notion of 'boundary central charge', as in [15]. We notice that the natural generalization of this concept for our set up is as a measure of the effective volume left after the geometrical capping.

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